Physics 321
Spring 2007
Due: 7 May 2007

**Assignment # 1**

1. Taylor 1.13

2. Taylor 1.22

3. In spherical polar coordinates, the basis vectors $\{\mathbf{e}_r, \mathbf{e}_\theta, \mathbf{e}_\phi\}$ can be expressed in terms of the Cartesian basis vectors as

$$\begin{aligned} \mathbf{e}_r &= \sin\theta\cos\phi\,\mathbf{e}_x + \sin\theta\sin\phi\,\mathbf{e}_y + \cos\theta\,\mathbf{e}_z \\ \mathbf{e}_\theta &= \cos\theta\cos\phi\,\mathbf{e}_x + \cos\theta\sin\phi\,\mathbf{e}_y - \sin\theta\,\mathbf{e}_z \\ \mathbf{e}_\phi &= -\sin\phi\,\mathbf{e}_x + \cos\phi\,\mathbf{e}_y \end{aligned}$$

   Assume that the coordinates $(r, \theta, \phi)$ are functions of time $t$. (Spherical polar coordinates are discussed in Taylor pp. 134–138.)

   (a) Find expressions for the time derivatives of $\{\mathbf{e}_r, \mathbf{e}_\theta, \mathbf{e}_\phi\}$.

   (b) A particle of mass $m$ is subject to a force $\mathbf{F}$. The motion of the particle is described in spherical coordinates. Find the components of the force

$$\mathbf{F} = F_r\,\mathbf{e}_r + F_\theta\,\mathbf{e}_\theta + F_\phi\,\mathbf{e}_\phi$$

   in terms of $(r, \theta, \phi)$ and their time derivatives.

4. Verify the following vector identities for arbitrary vectors $\mathbf{a}$, $\mathbf{b}$, and $\mathbf{c}$. $\phi$ is an arbitrary function. The Einstein summation convention must be used on this problem.

   (a) $\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) = \mathbf{b}(\mathbf{a} \cdot \mathbf{c}) - \mathbf{c}(\mathbf{a} \cdot \mathbf{b})$

   (b) $\nabla \times (\phi \nabla \phi) = 0$

   (c) $\nabla \times (\nabla \times \mathbf{a}) = \nabla(\nabla \cdot \mathbf{a}) - \nabla^2 \mathbf{a}$

   (d) $\nabla \cdot (\mathbf{a} \times \mathbf{b}) = (\nabla \times \mathbf{a}) \cdot \mathbf{b} - (\nabla \times \mathbf{b}) \cdot \mathbf{a}$

5. Expand each of the following expressions for small $\epsilon$ and keep three terms:

   (a) $(1 + \epsilon\omega_1 + \epsilon^2\omega_2)^{-2}$

   (b) $\sqrt{1 - \frac{1}{2}\epsilon^2 t - \frac{1}{8}\epsilon^4 t}$

   (c) $\sin(s + \epsilon\omega_1 s + \epsilon^2\omega_2 s)$

6. Use second-order perturbation theory to find approximations to the roots of the following equations. Compare your results with numerical solutions.

   (a) $x^3 - \epsilon x - 1 = 0$.

   (b) $x^3 - \epsilon x^2 - x = 0$.

7. Consider the equation

$$y' + y = \epsilon y^2, \qquad y(0) = 1.$$

   (a) Determine a three-term expansion for small $\epsilon$.

   (b) Find the exact solution.

   (c) Expand the exact solution in (b) for small $\epsilon$ and compare the result with (a).

8. Write a short essay to introduce yourself. You might write about your interests, hobbies, family, significant experiences, future plans, etc. Include a list of the physics and mathematics courses you have either taken or are currently enrolled in. Your response should be about one page. (This essay will be turned in seperately from the other problems on this assignment, i.e., do not staple the essay to the other problems).