Homework Assignments Physics 441

This is a first draft; it may be revised.

Assignment 1

122 Review: 1-8;

1.2, 1.3, 1.5, 1.7, 1.10, 1.12, 1.13, 1.14, 1.16, 1.18, 1.20.

1.100: Consider a charged rod of length b with uniform linear charge density $\lambda = Q/b$, as shown below. Find the electric field components $E_x(x)$ and $E_y(x)$ for points along the x-axis. Work out the integrals completely using Maple.

Assignment 2

1.26, 1.27, 1.28, 1.31, 1.32, 1.33, 1.36, 1.37, 1.38, 1.42, 1.43, 1.45

1.101: Find the names and dates for: the divergence theorem (Gauss's law). (Find all 4 names.)

1.102: Find the names and dates for: the curl (Stokes) theorem.

1.103: Consider a parabolic rod with uniform linear charge density λ, as shown below. The formula for the points along the rod is $y = x^2/b$ with x in the interval $[0, b]$. (a) Find an integral expression for the total charge on the rod. Your answer should be a definite integral in x with proper limits. You do not have to work out the integral. (b) Find integral expressions for $E_x(y)$ and $E_y(y)$ along the y-axis. Your answers should be definite integrals in x with proper limits. (c) Finally, use Maple to make plots of E_x and E_y from $y = 0.1b$ to $y = 10b$. To make the plots use $\lambda = 1$, $\frac{1}{4\pi\epsilon_0} = 1$, and $b = 1$.

Assignment 3

1.52, 1.60, 1.62

Physics 122 Review: 9-15

2.1, 2.3, 2.5, 2.6, 2.7

1.104 Consider a semi-circular rod of radius a lying in the xy-plane as shown below. The rod is charged non-uniformly with charge density given by $\lambda(\phi) = \lambda_o \phi^2$, with ϕ the usual polar angle in cylindrical coordinates. (a) Find the total charge on the rod. Work out the integral completely. (b) Find integral expressions for $E_x(z)$, $E_y(z)$, and $E_z(z)$ along the z-axis. Work out the integrals completely and make plots of E_y and E_z from $z = 0$ to $z = 10a$.

Assignment 4

Physics 122 Review: 16-24

2.9, 2.10, 2.11, 2.12, 2.14, 2.16, 2.17, 2.20, 2.21, 2.24, 2.28, 2.29

1.105 Consider a uniformly charged curved spiral of wire lying on a sphere of radius a with constant linear charge density λ. (For what it is worth, a perspective plot of this curved spiral is shown below.) The equation for the position of the wire in spherical coordinates is $\phi = 4\theta$, where θ and ϕ are the polar and azimuthal angles in spherical coordinates. As usual, θ ranges from 0 to π. (a) Find the total charge on the wire. You will encounter a hard definite integral. Find its value numerically using Maple. (b) Find integral expressions for E_x, E_y, and E_z at the origin. Use Maple to obtain numerical answers for these three fields with each one given in the form

$$E_? = f\frac{\lambda}{\epsilon_0 a}$$

where f is the numerical factor to be found with Maple. If any components turn out to be zero show how you could have gotten this answer just by examining the form of the integrand.

Assignment 5

Physics 122 Review: 25-33

2.30, 2.31, 2.32,.2.35, 2.39, 2.44, 2.48

2.100: Find the names and dates for: earliest electrostatic experiment.

2.101: Find the names and dates for: 2 kinds of electricity.

2.102: Find the names and dates for: "positive" and "negative" electricity.

2.103: Find the names and dates for: animal electricity (birth and death).

2.104: Find the names and dates for: principle of charge conservation.

2.105: Find the names and dates for: inverse-square law for electricity.

Assignment 6

Physics 122 Review: 34-40

3.2, 3.3

2.106: Find the names and dates for: $\nabla^2 V = 0$.

2.107: Find the names and dates for: $\nabla^2 V = -\rho/\epsilon_0$.

2.108: Find the names and dates for: $E = \sigma/\epsilon_0$ on a conductor.

2.109: Consider a charged sphere of radius a with non-uniform charge density $\rho(r) = \rho_o(r/a)^3$. (a) Find the radial electric field $E(r)$ both inside and outside of the sphere. (b) Find the potential $V(r)$ both inside and outside the sphere assuming that $V(\infty) = 0$.

Computation Problem 1: Consider a ring of charge of radius a and linear charge density λ (charge per unit length) centered at the origin of coordinates and lying in the xy-plane. (a) Find an integral expression for the potential $V(r, \theta)$ in spherical coordinates valid for all r and θ. (b) Rearrange the integral so that it is dimensionless, i.e., don't let r appear by itself, but only in the combination $s = r/a$. Then use Maple to find a numerical value for $V(a, \pi/4)$. Your answer should be a number multiplying $\lambda/(4\pi\epsilon_o)$. (c) Have Maple take the gradient of your integral expression for $V(r, \theta)$ to obtain integral expressions for E_r and E_θ. You can either use the *grad* command or *diff*. Numerically evaluate them at $(a, \pi/4)$.

Assignment 7

Physics 122 Review: 41-49

3.7, 3.8, 3.10, 3.11, 3.15

3.100: Consider an infinite slab of charge with uniform charge density ρ_o and thickness d, as shown below. Use Poisson's equation in the single variable x to find the potential V and electric field E everywhere subject to the boundary conditions that $V(0) = 0$ and $\mathbf{E}(-\infty) = E_o\mathbf{i}$.

2.110: Find the names and dates for: electricity flows in wires.

2.111: Find the names and dates for: tongue-test of a crude battery.

2.112: Find the names and dates for: fluorescent bulb.

2.113: Find the names and dates for: Use of correct boundary conditions on **E** and **B** with Maxwell's equations to derive optical results.

Computation Problem 2: Make contour and 3-d plots of the potential function $V(x, y)$ given by Griffiths for his Example 3.4 (p. 132) in Eq. (3.42). In this equation use $a = 2$, $b = 2$, and $V_0 = 1$. Experiment with different numbers of terms to keep in the sum and comment on how the plots change as more terms are kept. Do the sum and the plots with Maple. You should know how to do this from Physics 318; you can get some extra help from the Physics 230 disk hanging on the side of the cabinet in N212.

Assignment 8

Physics 122 Review: 50-58

3.18, 3.21, 3.23, 3.24

2.114: Find the names and dates for: capacitance and resistance.

2.115: Find the names and dates for: $R \propto \ell/A$.

2.116 Find the names and dates for: $V = IR$.

2.117: Find the names and dates for: dielectric constant.

Computation Problem 3: Do Example 4 (p. 132) in Griffiths using a grid instead of a Fourier sum. Use the method of Successive Over-Relaxation (SOR) as discussed in the course outline. You can download MATLAB code from the Physics 441 website that is ready to do this problem. When you run it set $a = 2$, $b = 2$, and $V_0 = 1$. Run it to convergence and use your grid solution to find $V(0, a/2)$. You should get about 0.11 when you use $N_x = N_y = 21$, any value of ω that works, and an error criterion of $\epsilon = 10^{-4}$. (a) Now experiment with the code by changing N_x and N_y, the error criterion ϵ and the overrelaxation parameter ω. Discuss how the code behaves as you change these parameters. (b) Compare your grid answer to the answer you get by numerically performing the sum in the solution given in Eq. (3.42) in the text. Keep enough terms in the sum that you get an accurate answer. Explain why the grid answer and the Fourier sum answer aren't exactly the same. (c) Change the boundary conditions in the code so that $V = 0$ on each edge of the boundary except at $x = -a$. Run the code again and find the new value of $V(0, a/2)$.

Assignment 9

Physics 122 problems: 59-67

3.26, 3.27, 3.33, 3.39

Computation Problem 4: Modify the code you worked with in Problem 3 so that it can handle the case of a rectangular pipe with a grounded diagonal plate from $(-b, 0)$ to (b, a). Let V on the edges all be zero except at $x = b$ where it is $V_0 = 1$. To do this case you will need to load the grid points along this diagonal with zeroes and include some logic in the SOR loop to keep the code from changing V on these diagonal points. The easiest way to do this is to just load V full of zeroes to start with and then only let SOR modify V at points within the triangular region bounded by the grounded bottom, the grounded diagonal, and the right side at V_0. This is easily accomplished by changing the for-loop structure from this

```
for i=2:(Nx-1)    % Loop over interior points only
   for j=2:(Ny-1)
```

to this:

```
for i=3:(Nx-1)    % Loop over interior points only
   for j=2:??
```

Hand in a 3-d graph of the potential showing how $V(x, y)$ is stretched over this triangular region.

Note that the i-loop has to start at 3 instead of at 2 because the only j-points in the triangle at $i = 2$ are the grounded bottom and the grounded diagonal. Also note that ?? means that you are supposed to figure out what to put in this position. The j-loop is moving the V-index vertically in y, and you want it it to stop just before it reaches the grounded diagonal. Hint: ?? involves i.

In case you want to use them, the valid MATLAB logic tests are: ==, <, >, <=, >=, and ~= for equal, less than, greater than, less than or equal, greater than or equal, and not equal. Type *help if* to see how to write if statements in MATLAB.

Assignment 10

4.1, 4.4, 4.6, 4.7, 4.8, 4.10, 4.11, 4.13

Computation Problem 5: Modify the code you worked with in Problem 3 so that it can handle the same geometry and boundary conditions, but this time there is a uniform charge density ρ_o present throughout the rectangular pipe. Again, use $a = 2$, $b = 2$, and $V_0 = 1$. Experiment with different values of ρ_o ranging from 10^{-12}. to 10^{-9} by powers of 10. Explain what happens to the saddle point in V and tell why it changes as charge is added. For the case $\rho_o = 10^{-10}$ give the value of V at $(0, a/2)$.

Assignment 11

4.15, 4.16, 4.17, 4.18, 4.19, 4.22, 4.26, 4.28, 4.38, 4.39

Assignment 12

Physics 122 Review: 68-74

5.1, 5.2 [add part (d): $\mathbf{v}(0) = 2(\mathrm{E/B})\ (\mathbf{j} + \mathbf{k})$],
5.6, 5.8, 5.9, 5.11, 5.12, 5.13, 5.14, 5.16, 5.18

5.100: Find the names and dates for: naturally occurring magnetism

5.101: Find the names and dates for: $\oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 I$.

5.102: Find the names and dates for: electrical current twists a compass needle.

5.103: Find the names and dates for: vector potential $\mathbf{A}$

5.104: Find the names and dates for: $\mathbf{B}, \mathbf{M}$, and $\mathbf{H}$.

Computation Problem 6: Consider a circular loop of current of radius a and carrying current I, centered at the origin of coordinates and lying in the xy-plane. (a) Find integral expressions for both $B_r(r, \theta)$ and $B_\theta(r, \theta)$ in spherical coordinates, valid for all r and θ. Maple may be helpful in getting the correct integrals. (b) Rearrange the integral so it is dimensionless, i.e., don't let r appear by itself but only as r/a. Then use Maple to find $B_r(a, \pi/4)$ and $B_\theta(a, \pi/4)$. Your answer should be a number multiplying $\mu_o I/4\pi$. (c) Use your calculation in part (b) to make a Maple plot of the magnetic field in the plane of the loop (at $\theta = \pi/2$) from $r = 0$ out to $r = 2a$. Include the direction (sign) of B in your plot and make sure you include enough integration points to get accurate results. In particular, make sure you get the correct answer at the center of the loop where an analytic result is available.

Assignment 13

Physics 122 Review: 75-84

5.20, 5.22, 5.23, 5.29, 5.32, 5.33, 5.36, 5.39

5.101 In the analogy between electrostatics and magnetostatics which magnetic quantities are most analogous to these electrical ones (briefly explain your answers). (a) $\mathbf{E}$, (b) V, (c) ρ, (d) ϵ_0, (e) $\mathbf{P}$, (f) $\mathbf{D}$, (g) σ, (h) λ

Assignment 14

Physics 122 Review: 85-97

6.1, 6.3, 6.6, 6.7, 6.8, 6.10, 6.12, 6.13, 6.14