Important Stuff From Physics 441

Fundamental Constants:

$\epsilon_o = 8.85 \times 10^{-12}\mathrm{C}^2/\mathrm{N} \cdot m^2 \quad \mu_o = 4\pi \times 10^{-7}\mathrm{N/A}^2 \quad c = 3.00 \times 10^8\mathrm{m/sec} \quad e = 1.60 \times 10^{-19}\mathrm{C} \quad m_e = 9.11 \times 10^{-31}\mathrm{kg}$

Chapter 1

Most important vector in E&M:

$$\text{Vector from } \mathbf{r}' \text{ to } \mathbf{r}: \quad \mathbf{r} - \mathbf{r}' \quad ; \quad |\mathbf{r} - \mathbf{r}'| = \sqrt{r^2 + r'^2 - 2\mathbf{r} \cdot \mathbf{r}'} = \sqrt{r^2 + r'^2 - 2rr'\cos\theta} \quad .$$

Taylor Expansion:

$$f(a+x) = f(a) + f'(a)x + \frac{1}{2}f''(a)x^2 + \ldots \frac{1}{n!}f^{(n)}(a)x^n + \ldots$$

Vector derivatives (gradient, divergence, and curl):

$$\nabla f \quad ; \quad \nabla \cdot \mathbf{F} \quad ; \quad \nabla \times \mathbf{F} \quad : \quad \text{know what they mean.}$$

Laplacian Operator:

$$\nabla^2 f = \nabla \cdot \nabla f$$

Vector Identities:

(1)
$$\mathbf{A} \cdot (\mathbf{B} \times \mathbf{C}) = \mathbf{B} \cdot (\mathbf{C} \times \mathbf{A}) = \mathbf{C} \cdot (\mathbf{A} \times \mathbf{B})$$

(2)
$$\mathbf{A} \times (\mathbf{B} \times \mathbf{C}) = \mathbf{B}(\mathbf{A} \cdot \mathbf{C}) - \mathbf{C}(\mathbf{A} \cdot \mathbf{B})$$

(3)
$$\nabla(fg) = f(\nabla g) + g(\nabla f)$$

(4)
$$\nabla(\mathbf{A} \cdot \mathbf{B}) = \mathbf{A} \times (\nabla \times \mathbf{B}) + \mathbf{B} \times (\nabla \times \mathbf{A}) + (\mathbf{A} \cdot \nabla)\mathbf{B} + (\mathbf{B} \cdot \nabla)\mathbf{A}$$

(5)
$$\nabla \cdot (f\mathbf{A}) = f\nabla \cdot \mathbf{A} + \mathbf{A} \cdot (\nabla f)$$

(6)
$$\nabla \cdot (\mathbf{A} \times \mathbf{B}) = \mathbf{B} \cdot (\nabla \times \mathbf{A}) - \mathbf{A} \cdot (\nabla \times \mathbf{B})$$

(7)
$$\nabla \times (f\mathbf{A}) = f(\nabla \times \mathbf{A}) - \mathbf{A} \times (\nabla f)$$

(8)
$$\nabla \times (\mathbf{A} \times \mathbf{B}) = (\mathbf{B} \cdot \nabla)\mathbf{A} - (\mathbf{A} \cdot \nabla)\mathbf{B} + \mathbf{A}(\nabla \cdot \mathbf{B}) - \mathbf{B}(\nabla \cdot \mathbf{A})$$

Second derivatives:

(9)
$$\nabla \cdot (\nabla \times \mathbf{A}) = 0$$

(10)
$$\nabla \times \nabla f = 0$$

(11)
$$\nabla \times (\nabla \times \mathbf{A}) = \nabla(\nabla \cdot \mathbf{A}) - \nabla^2 \mathbf{A}$$

Integral theorems:

$$\text{Gradient}: \quad \int_{\mathbf{r}_a}^{\mathbf{r}_b} \nabla f \cdot d\mathbf{l} = f(\mathbf{r}_b) - f(\mathbf{r}_a)$$

$$\text{Divergence Theorem (Gauss's Law)}: \quad \int_V \nabla \cdot \mathbf{F} \, dV = \oint_S \mathbf{F} \cdot \mathbf{da}$$

$$\text{Stokes Theorem (Ampere's and Faraday's Laws)}: \quad \int_S \nabla \times \mathbf{F} \cdot \mathbf{da} = \oint_C \mathbf{F} \cdot d\mathbf{l}$$

Spherical coordinates:

$$
\begin{aligned}
x &= r\sin\theta\cos\phi & \hat{\imath} &= \sin\theta\cos\phi\;\hat{r} + \cos\theta\cos\phi\;\hat{\theta} - \sin\phi\;\hat{\phi} \\
y &= r\sin\theta\sin\phi & \hat{\jmath} &= \sin\theta\sin\phi\;\hat{r} + \cos\theta\sin\phi\;\hat{\theta} + \cos\phi\;\hat{\phi} \\
z &= r\cos\theta & \hat{k} &= \cos\theta\;\hat{r} - \sin\theta\;\hat{\theta}
\end{aligned}
$$

$$
\begin{aligned}
r &= \sqrt{x^2+y^2+z^2} & \hat{r} &= \sin\theta\cos\phi\;\hat{\imath} + \sin\theta\sin\phi\;\hat{\jmath} + \cos\theta\;\hat{k} \\
\theta &= \tan^{-1}\left(\sqrt{x^2+y^2}/z\right) & \hat{\theta} &= \cos\theta\cos\phi\;\hat{\imath} + \cos\theta\sin\phi\;\hat{\jmath} - \sin\theta\;\hat{k} \\
\phi &= \tan^{-1}(y/x) & \hat{\phi} &= -\sin\phi\;\hat{\imath} + \cos\phi\;\hat{\jmath}
\end{aligned}
$$

Calculus in spherical coordinates:

Line and volume elements:

$$d\mathbf{l} = dr\hat{r} + rd\theta\hat{\theta} + r\sin\theta d\phi\hat{\phi} \qquad d\tau = r^2\sin\theta dr d\theta d\phi$$

Gradient:

$$\nabla f = \frac{\partial f}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial f}{\partial \theta}\hat{\theta} + \frac{1}{r\sin\theta}\frac{\partial f}{\partial \phi}\hat{\phi}$$

Divergence:

$$\nabla\cdot\mathbf{v} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 v_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta v_\theta) + \frac{1}{r\sin\theta}\frac{\partial v_\phi}{\partial\phi}$$

Curl:

$$\nabla\times\mathbf{v} = \frac{1}{r\sin\theta}\left[\frac{\partial}{\partial\theta}(\sin\theta v_\phi) - \frac{\partial v_\theta}{\partial\phi}\right]\hat{r} + \frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial v_r}{\partial\phi} - \frac{\partial}{\partial r}(r v_\phi)\right]\hat{\theta} + \frac{1}{r}\left[\frac{\partial}{\partial r}(r v_\theta) - \frac{\partial v_r}{\partial\theta}\right]\hat{\phi}$$

Laplacian of a Scalar Field:

$$\nabla^2 f = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial f}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial f}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 f}{\partial\phi^2}$$

Laplacian of a Vector Field:

$$\nabla^2\mathbf{A} = \left(\nabla^2 A_r - \frac{2A_r}{r^2} - \frac{2}{r^2}\frac{\partial A_\theta}{\partial\theta} - \frac{2\cot\theta A_\theta}{r^2} - \frac{2}{r^2\sin\theta}\frac{\partial A_\phi}{\partial\phi}\right)\hat{r} +$$

$$\left(\nabla^2 A_\theta + \frac{2}{r^2}\frac{\partial A_r}{\partial\theta} - \frac{A_\theta}{r^2\sin^2\theta} - \frac{2\cos\theta}{r^2\sin^2\theta}\frac{\partial A_\phi}{\partial\phi}\right)\hat{\theta} + \left(\nabla^2 A_\phi - \frac{A_\phi}{r^2\sin^2\theta} + \frac{2}{r^2\sin\theta}\frac{\partial A_r}{\partial\phi} + \frac{2\cos\theta}{r^2\sin^2\theta}\frac{\partial A_\theta}{\partial\phi}\right)\hat{\phi}$$

Cylindrical coordinates:

$$\begin{aligned} x &= s\cos\phi & \hat{i} &= \cos\phi\ \hat{s} - \sin\phi\ \hat{\phi} \\ y &= s\sin\phi & \hat{j} &= \sin\phi\ \hat{s} + \cos\phi\ \hat{\phi} \\ z &= z & \hat{k} &= \hat{k} \end{aligned}$$

$$\begin{aligned} s &= \sqrt{x^2+y^2} & \hat{s} &= \cos\phi\ \hat{i} + \sin\phi\ \hat{j} \\ \phi &= \tan^{-1}(y/x) & \hat{\phi} &= -\sin\phi\ \hat{i} + \cos\phi\ \hat{j} \\ z &= z & \hat{k} &= \hat{k} \end{aligned}$$

Calculus in cylindrical coordinates:

Line and volume elements:

$$d\mathbf{l} = ds\hat{s} + sd\phi\hat{\phi} + dz\hat{k} \qquad\qquad d\tau = sdsd\phi dz$$

Gradient:

$$\nabla f = \frac{\partial f}{\partial s}\hat{s} + \frac{1}{s}\frac{\partial f}{\partial \phi}\hat{\phi} + \frac{\partial f}{\partial z}\hat{k}$$

Divergence:

$$\nabla\cdot\mathbf{v} = \frac{1}{s}\frac{\partial}{\partial s}(sv_s) + \frac{1}{s}\frac{\partial v_\phi}{\partial \phi} + \frac{\partial v_z}{\partial z}$$

Curl:

$$\nabla\times\mathbf{v} = \left[\frac{1}{s}\frac{\partial v_z}{\partial \phi} - \frac{\partial v_\phi}{\partial z}\right]\hat{s} + \left[\frac{\partial v_s}{\partial z} - \frac{\partial v_z}{\partial s}\right]\hat{\phi} + \frac{1}{s}\left[\frac{\partial}{\partial s}(sv_\phi) - \frac{\partial v_s}{\partial \phi}\right]\hat{k}$$

Laplacian of a Scalar Field:

$$\nabla^2 f = \frac{1}{s}\frac{\partial}{\partial s}\left(s\frac{\partial f}{\partial s}\right) + \frac{1}{s^2}\frac{\partial^2 f}{\partial \phi^2} + \frac{\partial^2 f}{\partial z^2}$$

Laplacian of a Vector Field:

$$\nabla^2\mathbf{A} = \left(\nabla^2 A_s - \frac{A_s}{s^2} - \frac{2}{s^2}\frac{\partial A_\phi}{\partial \phi}\right)\hat{s} + \left(\nabla^2 A_\phi + \frac{2}{s^2}\frac{\partial A_s}{\partial \phi} - \frac{A_\phi}{s^2}\right)\hat{\phi} + \nabla^2 A_z\hat{z}$$

Chapter 2

Electric field is made by electric charge:

$$\mathbf{E}(\mathbf{r}) = \sum_i \frac{q_i(\mathbf{r}-\mathbf{r}_i)}{4\pi\epsilon_o|\mathbf{r}-\mathbf{r}_i|^3} + \int_C \frac{(\mathbf{r}-\mathbf{r}')\lambda(\mathbf{r}')dl'}{4\pi\epsilon_o|\mathbf{r}-\mathbf{r}'|^3} + \int_S \frac{(\mathbf{r}-\mathbf{r}')\sigma(\mathbf{r}')da'}{4\pi\epsilon_o|\mathbf{r}-\mathbf{r}'|^3} + \int_V \frac{(\mathbf{r}-\mathbf{r}')\rho(\mathbf{r}')d\tau'}{4\pi\epsilon_o|\mathbf{r}-\mathbf{r}'|^3}$$

Gauss's Law:

$$\text{Integral form}: \quad \oint \mathbf{E}\cdot\mathbf{n}\, da = \frac{q_{enc}}{\epsilon_o} \quad ; \quad \text{Differential form}: \quad \nabla\cdot\mathbf{E} = \frac{\rho}{\epsilon_o}$$

Curl of $\mathbf{E}$:

$$\text{Integral form}: \quad \oint \mathbf{E}\cdot \mathrm{d}\mathbf{l} = 0 \quad ; \quad \text{Differential form}: \quad \nabla\times\mathbf{E} = 0$$

Electrostatic potential (voltage) is made by electric charge:

$$V(\mathbf{r}) = \sum_i \frac{q_i}{4\pi\epsilon_o|\mathbf{r}-\mathbf{r}_i|} + \int_C \frac{\lambda(\mathbf{r}')dl'}{4\pi\epsilon_o|\mathbf{r}-\mathbf{r}'|} + \int_S \frac{\sigma(\mathbf{r}')da'}{4\pi\epsilon_o|\mathbf{r}-\mathbf{r}'|} + \int_V \frac{\rho(\mathbf{r}')d\tau'}{4\pi\epsilon_o|\mathbf{r}-\mathbf{r}'|}$$

Boundary conditions on $\mathbf{E}$ and V at a surface:

$$V \text{ is continuous}$$

$$\text{Tangential component}: \quad E_\parallel \text{ is continuous} \quad \text{Normal component}: \quad \Delta E_\perp = \frac{\sigma}{\epsilon_o}$$

Force, electric field, and potential energy:

$$\mathbf{F} = q\mathbf{E} \quad ; \quad \mathbf{E} = -\nabla V \quad ; \quad U = qV \quad ; \quad \mathbf{F} = -\nabla U$$

Potential Energy:

$$U = \sum_{pairs} \frac{q_i q_j}{4\pi\epsilon_o r_{ij}} \quad = \quad \frac{1}{2}\sum q_j V_j \quad = \quad \frac{1}{2}\int_V \rho V d\tau$$

Energy Density:

$$u(\mathbf{r}) = \frac{\epsilon_o}{2}\mathbf{E}^2 \quad ; \quad U = \int_{all\ space} u(\mathbf{r})d\tau$$

Capacitance:

$$Q = C\Delta V \quad ; \quad U = \frac{1}{2}C\Delta V^2 \quad ; \quad C = \frac{\epsilon A}{d} \quad \text{(parallel plate)}$$

$$C = C_1 + C_2 + ... \quad \text{(parallel)} \quad ; \quad \frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + ... \quad \text{(series)}$$

Chapter 3

Solving Laplace's and Poisson's Equations:

$$\nabla^2 V = 0 \quad ; \quad \nabla^2 V = -\frac{\rho}{\epsilon_o}$$

Know the uniqueness and superposition theorems for Laplace.

With one independent variable, just integrate twice:

$$\text{Cartesian}: \frac{d^2V}{dx^2} = 0 \quad ; \quad \text{Cylindrical}: \frac{1}{s}\frac{d}{ds}\left(s\frac{dV}{ds}\right) = 0 \quad ; \quad \text{Spherical}: \frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dV}{dr}\right) = 0$$

With two independent variables, use separable solutions:

Cartesian (x, y):

x-functions	y-functions
$\cos px$, $\sin px$	$\cosh py$, $\sinh py$, $e^{\pm py}$
$\cosh px$, $\sinh px$, $e^{\pm px}$	$\cos py$, $\sin py$

Cylindrical (s, ϕ):

m	ϕ-functions	s-functions
0	1	$\ln s$ (charged wire) ; 1 (additive constant)
1	$\cos\phi$, $\sin\phi$	$1/s$ (dipole) ; s (uniform field)
2	$\cos 2\phi$, $\sin 2\phi$	$1/s^2$ (quadrupole) ; s^2 (external quadupole)
.	.	.
.	.	.
m	$\cos m\phi$, $\sin m\phi$	$1/s^m$; s^m

Spherical (r, θ):

n	θ-functions	r-functions
0	1	$1/r$ (point charge) ; 1 (additive constant)
1	$P_1(\cos\theta) = \cos\theta$	$1/r^2$ (dipole) ; r (uniform field)
2	$P_2(\cos\theta) = \frac{1}{2}(3\cos^2\theta - 1)$	$1/r^3$ (quadrupole) ; r^2 (external quadupole)
.	.	.
.	.	.
n	$P_n(\cos\theta)$	$1/r^{n+1}$; r^n

Useful Trig Identities:

$$\cos^2\theta + \sin^2\theta = 1$$

$$\cos 2\theta = \cos^2\theta - \sin^2\theta \quad ; \quad \sin 2\theta = 2\cos\theta\sin\theta$$

$$\cos^2\theta = \frac{1+\cos 2\theta}{2} \quad ; \quad \sin^2\theta = \frac{1-\cos 2\theta}{2}$$

$$\cos 3\theta = -3\cos\theta + 4\cos^3\theta \quad ; \quad \sin 3\theta = 3\sin\theta - 4\sin^3\theta$$

$$\cos 4\theta = 8\cos^4\theta - 8\cos^2\theta + 1 \quad ; \quad \sin 4\theta = 8\cos^3\theta\sin\theta - 4\cos\theta\sin\theta \quad ;$$

Image Charge Method:

Add the potentials of point charges to build equipotential planes, spheres, and cylinders.

Plane:
For an infinite conducting plane and a point charge (or a charged wire) a distance d above it, get the total V by using an image of opposite sign but equal magnitude located a distance d below the plane.

Cylinder:
For a conducting cylinder of radius a and a charged wire with linear charge density λ a distance d away from the center of the cylinder, the total V can be computed by using an image wire with $\lambda' = -\lambda$ located a distance $b = a^2/d$ away from the center of the cylinder, toward the charged wire on the outside.

Sphere:
For a grounded conducting sphere of radius a and a point charge q a distance d away from the center of the sphere, the total V can be computed by using an image charge with $q' = -aq/d$ located a distance $b = a^2/d$ away from the center of the sphere, toward the point charge on the outside.

Electric Dipoles:

$$\mathbf{p} = q\mathbf{l} \quad \text{(from} \quad - \quad \text{to} \quad + \quad) \;=\; \int \mathbf{r}\rho(\mathbf{r})d\tau$$

Dipole electrostatic potential and field:

$$V(\mathbf{r}) = \frac{\mathbf{p}\cdot\mathbf{r}}{4\pi\epsilon_o r^3} \quad ; \quad \mathbf{E}(\mathbf{r}) = \frac{(3(\mathbf{p}\cdot\hat{r})\hat{r} - \mathbf{p})}{4\pi\epsilon_o r^3}$$

Dipole energy, torque, and force:

$$U = -\mathbf{p}\cdot\mathbf{E}_{external} \quad ; \quad \mathbf{N} = \mathbf{p}\times\mathbf{E}_{external} \quad ; \quad \mathbf{F} = (\mathbf{p}\cdot\nabla)\mathbf{E}_{external}$$

Chapter 4

Polarization:

$$\mathbf{P} = \frac{\Delta \mathbf{p}}{\Delta V}$$

Polarization Charge Densities:

$$\rho_p = -\nabla \cdot \mathbf{P} \quad ; \quad \sigma_p = \mathbf{P} \cdot \mathbf{n}$$

$\mathbf{D}$, χ_e, ϵ, K:

General:

$$\mathbf{D} = \epsilon_o \mathbf{E} + \mathbf{P} \quad ; \quad \nabla \cdot \mathbf{D} = \rho_f$$

Linear media:

$$\mathbf{P} = \epsilon_0 \chi_e \mathbf{E} \quad ; \quad \mathbf{D} = \epsilon \mathbf{E} \quad ; \quad \epsilon = \epsilon_o (1 + \chi_e) \quad ; \quad \epsilon = K \epsilon_o$$

Boundary Conditions ($\mathbf{n}_2$ points into medium 2):

$$V_2 - V_1 = 0 \quad ; \quad (\text{same as} \quad E_{2\parallel} - E_{1\parallel} = 0) \quad ; \quad \text{normal component :} \quad (\mathbf{D}_2 - \mathbf{D}_1) \cdot \mathbf{n}_2 = \sigma_f \tag{1}$$

Energy density in linear dielectrics:

$$u(\mathbf{r}) = \frac{1}{2} \mathbf{D} \cdot \mathbf{E}$$

Forces and Torques:

$$F_x = -\frac{\partial U}{\partial x} \quad , \quad N = -\frac{\partial U}{\partial \theta} \quad (\text{constant Q}) \quad ; \quad F_x = +\frac{\partial U}{\partial x} \quad , \quad N = +\frac{\partial U}{\partial \theta} \quad (\text{constant V})$$

Molecular Field and Polarizability:

$$\mathbf{p} = \alpha \mathbf{E} \qquad \mathbf{P} = \frac{\mathbf{\Delta p}}{\Delta V} \qquad \mathbf{P} = N \alpha \mathbf{E}_{else}$$

Clausius-Mosotti:

$$\mathbf{P} = \frac{N\alpha}{(1 - N\alpha/3\epsilon_0)} \mathbf{E} \quad ; \quad \chi_e = \frac{N\alpha/\epsilon_0}{(1 - N\alpha/3\epsilon_0)} \quad ; \quad \alpha = \frac{3\epsilon_o (K-1)}{N(K+2)}$$

Chapter 5

Lorentz-Newton Force Law:

$$\mathbf{F} = m\mathbf{a} = m\frac{d\mathbf{v}}{dt} = m\frac{d^2\mathbf{r}}{dt^2} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

Particle Motion:

In a uniform magnetic field a particle moves at constant velocity along $\mathbf{B}$ and executes rotational motion in the plane perpendicular to $\mathbf{B}$ at cyclotron frequency ω_c and with Larmor radius r_L:

$$\omega_c = \frac{qB}{m} \quad ; \quad r_L = \frac{v_\perp}{\omega_c}$$

where $v_\perp$ means the magnitude of the particle velocity perpendicular to the magnetic field. If a uniform electric field $\mathbf{E}$ is also present then $E_\parallel$, the component of $\mathbf{E}$ along the magnetic field, accelerates the particle along the magnetic field line and $E_\perp$, the component perpendicular to $\mathbf{B}$, causes the particle to drift sideways at the $\mathbf{E} \times \mathbf{B}$ velocity:

$$\mathbf{v}_D = \frac{\mathbf{E} \times \mathbf{B}}{B^2}$$

Since the magnetic force is always perpendicular to $\mathbf{v}$, magnetic forces **do no work**. Hence, $\mathbf{B}$ can change a particle's direction, but can't speed it up or slow it down.

Current and Current Densities:

$$I = \frac{dQ}{dt} \quad ; \quad \mathbf{I} = \lambda\mathbf{v} \quad ; \quad \mathbf{K} = \frac{d\mathbf{I}}{d\ell_\perp} = \sigma\mathbf{v} \quad ; \quad \mathbf{J} = \frac{d\mathbf{I}}{da_\perp} = \rho\mathbf{v}$$

$$I = \int K dl_\perp \quad = \quad \int_S \mathbf{J} \cdot d\mathbf{a}$$

Magnetic Forces on Currents:

$$\mathbf{F} = I \int d\mathbf{l} \times \mathbf{B} = \int (\mathbf{K} \times \mathbf{B}) da = \int (\mathbf{J} \times \mathbf{B}) d\tau$$

Continuity Equation:

Charge is conserved, so if there is current flow into or out of some region, the charge density there must change:

$$\nabla \cdot \mathbf{J} = -\frac{\partial \rho}{\partial t}$$

Loop and Junction Laws, Resistors:

$$\sum \Delta V_j = 0 \quad ; \quad \sum I_j = 0 \quad ; \quad R = R_1 + R_2 + ... \text{ (series)} \quad ; \quad \frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + ... \text{ (parallel)}$$

Biot-Savart Law:

$$\mathbf{B}(\mathbf{r}) = \frac{\mu_o I}{4\pi} \oint_C \frac{d\mathbf{l}' \times (\mathbf{r} - \mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|^3} \quad = \quad \frac{\mu_o}{4\pi} \int_S \frac{\mathbf{K}(\mathbf{r}') \times (\mathbf{r} - \mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|^3} da' \quad = \quad \frac{\mu_o}{4\pi} \int_V \frac{\mathbf{J}(\mathbf{r}') \times (\mathbf{r} - \mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|^3} d\tau'$$

Maxwell's Equations (steady currents):

$$\nabla \cdot \mathbf{B} = 0 \quad ; \quad \nabla \times \mathbf{B} = \mu_o \mathbf{J}$$

Ampere's Law:

$$\oint_C \mathbf{B} \cdot d\mathbf{l} = \mu_o I_{enclosed} \quad ; \quad I_{enclosed} = \int_S \mathbf{J} \cdot d\mathbf{a}$$

Vector Potential:

$$\mathbf{B} = \nabla \times \mathbf{A} \quad ; \quad \mathbf{A}(\mathbf{r}) = \frac{\mu_o}{4\pi} \int_V \frac{\mathbf{J}(\mathbf{r}')d\tau'}{|\mathbf{r}-\mathbf{r}'|} \quad (\text{if} \ \ \nabla \cdot \mathbf{A} = 0, \ \ \text{Coulomb gauge})$$

Boundary conditions on **B** and **A** at a surface:

$$\mathbf{A} \ \ \text{is continuous}$$

Tangential to the surface there are two cases:

B parallel to **K** is continuous.

B perpendicular to **K** jumps

$$\Delta B_{tangent} = \mu_0 K$$

Use the right-hand rule on **K** to see which way the field jumps.

Perpendicular to the surface we have

$$B_\perp \ \ \text{is continuous}$$

These two statements can be combined into one if we let $\hat{n}$ be the normal to the surface pointing into the region **above** the surface and away from the region **below** it. Then

$$\mathbf{B}_{above} - \mathbf{B}_{below} = \mu_0 \mathbf{K} \times \hat{n}$$

Magnetic Dipoles:

$$\mathbf{A}(\mathbf{r}) = \frac{\mu_o}{4\pi} \frac{\mathbf{m} \times (\mathbf{r}-\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|^3} \quad ; \quad \mathbf{m} = I\mathbf{a} \quad = \quad \frac{1}{2} I \oint_C \mathbf{r} \times d\mathbf{l} \quad ; \quad \mathbf{N} = \mathbf{m} \times \mathbf{B}$$

$$U = -\mathbf{m} \cdot \mathbf{B} \quad ; \quad \mathbf{F} = \nabla(\mathbf{m} \cdot \mathbf{B}) = (\mathbf{m} \cdot \nabla)\mathbf{B}_{vacuum}$$

Pole Analogy:

North poles are "positive" and south poles are "negative." The "surface magnetic charge density" is given by $\sigma_m = \mathbf{M} \cdot \mathbf{n}$.

Chapter 6

Magnetization:

$$\mathbf{M} = \frac{\Delta \mathbf{m}}{\Delta V}$$

Magnetization Currents:

$$\mathbf{J}_b = \nabla \times \mathbf{M} \quad ; \quad \mathbf{K}_b = \mathbf{M} \times \mathbf{n}$$

H, χ_m, μ:

$$\mathbf{H} = \frac{1}{\mu_o}\mathbf{B} - \mathbf{M} \quad ; \quad \nabla \times \mathbf{H} = \mathbf{J}_f \quad ; \quad \mathbf{M} = \chi_m \mathbf{H} \quad ; \quad \mathbf{B} = \mu \mathbf{H} \quad ; \quad \mu = \mu_o(1+\chi_m)$$

Boundary Conditions ($\hat{n}$ points into the **above** region.)

$$\mathbf{H}^{\|}_{above} - \mathbf{H}^{\|}_{below} = \mathbf{K}_{free} \times \hat{n}$$

$$H^{\perp}_{above} - H^{\perp}_{below} = -(M^{\perp}_{above} - M^{\perp}_{below})$$